Further Maths Revision Paper 5

This paper consists of 5 questions covering CP1, CP2, FP1 and FM1.
(AS Further Maths: Q4 and Q5)

1

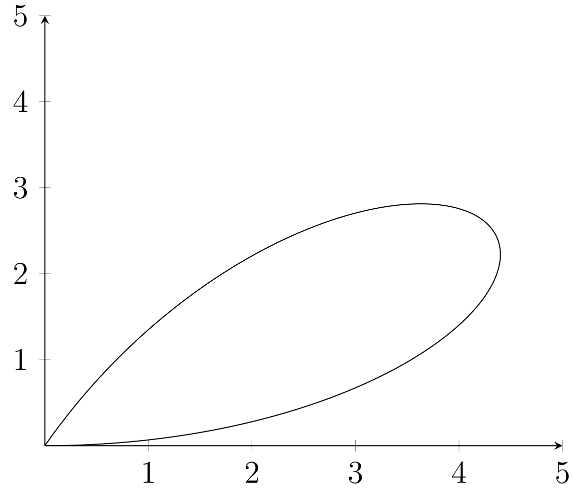


Figure 1

Figure 1 shows a section of the graph $r = 5\sin 3\theta$.
Find the area enclosed by the loop.

$\sin 3\theta = 0$

$3\theta = 0, \pi$

$\theta = 0, \frac{\pi}{3}$

$\frac{1}{2}\int_0^{\pi/3} 25\sin^2 3\theta \, d\theta$

$= \frac{25}{2}\int_0^{\pi/3} \sin^2 3\theta \, d\theta$

$\cos 6\theta = 1 - 2\sin^2 3\theta$

$2\sin^2 3\theta = 1 - \cos 6\theta$

$\sin^2 3\theta = \frac{1}{2} - \frac{\cos 6\theta}{2}$

$= \frac{25}{2}\int_0^{\pi/3} \frac{1}{2} - \cos\frac{6\theta}{2} \, d\theta$

$= \frac{25}{2}\left[\frac{1}{2}\theta - \frac{\sin 6\theta}{12}\right]_0^{\pi/3}$

$= \frac{25}{2}\left[\frac{\pi}{6} - 0\right]$

$= \frac{25\pi}{12}$

2

$$y = (1 + x^4)\sin x$$

Show that

$$\frac{\mathrm{d}^4 y}{\mathrm{d}x^4} = (x^4 - 72x^2 + 25)\sin x - 16x(x^2 - 6)\cos x$$

$u(x) = \sin x$ $\qquad$ $v(x) = (1 + x^4)$

$u'(x) = \cos x$ $\qquad$ $v'(x) = 4x^3$

$u''(x) = -\sin x$ $\qquad$ $v''(x) = 12x^2$

$u'''(x) = -\cos x$ $\qquad$ $v'''(x) = 24x$

$u''''(x) = \sin x$ $\qquad$ $v''''(x) = 24$

$$\frac{d^4y}{dx^2} = \underbrace{24\sin x}_{u\,v''''} + \underbrace{4\cos x(24x)}_{4u'v'''} - \underbrace{6(12x^2)\sin x}_{6u''v''}$$

$$\underbrace{- 4\cos x(4x^3)}_{4u'''v'} + (1+x^4)\sin x$$

$$= \sin x(24 - 72x^2 + 1 + x^4)$$

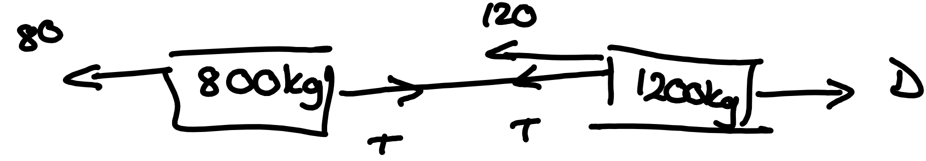
$$+ \cos x(96x - 16x^3)$$

3

A car of mass 1200kg tows another car of mass 800kg, the frictional resistances being 120N and 80N respectively.
If the tow rope has a breaking tension of 2000N find:

(a) the maximum possible acceleration.

(b) the maximum power the towing car can use at the instant when the speed is 10km/h



$T_{max} = 2000N$

a) $2000 - 80 = 800a$

$a = \frac{1920}{800} = 2.4 ms^{-2}$

b) $10 km/h = 10000 m/h$

$10000 m / 3600 s$

$= \frac{25}{9} ms^{-1}$

$D - 2120 = 1200(2.4)$

$D = 5000 N$

$H = Dv$

$= 5000 \times \frac{25}{9}$

$= 13889 W$

4

Given the differential equation

$$100\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 1 + (y-3)^2$$

with conditions $y = 4$ when $x = 0$ and $y = 4$ when $x = 1$
Use the approximation

$$y_{r+1} \approx 2y_r - y_{r-1} + h^2\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)_r$$

with $h = 1$ to find the value of y when $x = 4$

$y_0 = 4 \quad y_1 = 4$

$$\left(\frac{d^2y}{dx^2}\right)_0 = \frac{1+(4-3)^2}{100} = \frac{2}{100} = 0.02$$

$$\left(\frac{d^2y}{dx^2}\right)_1 = \frac{1+(4-3)^2}{100} = \frac{2}{100} = 0.02$$

$$y_2 \approx 2y_1 - y_0 + 1^2\left(\frac{d^2y}{dx^2}\right)_1$$

$$= 2(4)-(4) + 0.02$$

$$= 4.02$$

$$\left(\frac{d^2y}{dx^2}\right)_2 = \frac{1+(1.02)^2}{100} = 0.020404$$

$$y_3 \approx 2y_2 - y_1 + 1^2\left(\frac{d^2y}{dx^2}\right)_2$$

$$= 2(4.02)-(4) + 0.020404$$

$$= 4.060404$$

$$\left(\frac{d^2y}{dx^2}\right)_3 = \frac{1+(1.060404)^2}{100} = 0.0212\ldots$$

$$y_4 \approx 2y_3 - y_2 + 1^2\left(\frac{d^2y}{dx^2}\right)_3$$

$$= 2(4.060404) - 4.02 + 1^2(0.0212)$$

$$= \underline{\underline{4.12205}}$$

(a) Show that $\alpha = 3 + 2i$ is a root of $z^3 - 2z^2 - 11z + 52 = 0$.

(b) Hence find all the solutions of $z^3 - 2z^2 - 11z + 52 = 0$

a) $(3+2i)^3 - 2(3+2i)^2 - 11(3+2i) + 52$

$= 27 + 3(3)^2(2i) + 3(3)(2i)^2 + (2i)^3$

$-2(9 + 12i - 4) - 33 - 22i + 52$

$= 27 + 54i - 36 + -8i - 10 - 24i - 33 - 22i + 52$

$= \underline{\underline{0}}$

b) $(z - (3+2i))(z - (3-2i))$

$(z^2 - 6z + 13)(z + 4) = 0$

$\underline{\underline{z = 3+2i}}$ $\quad$ $\underline{\underline{z = 3-2i}}$ $\quad$ $\underline{\underline{z = -4}}$